esi® Engineering Bulletin
Electro Scientific Industries
7524 S. W. MACADAM AVE. • PORTLAND 19, OREGON

COMPUTERS
no. 16

APRIL 1960
REPLACES NOVEMBER 1959

APPLICATIONS OF THE esiac ALGEBRAIC COMPUTER

1. Frequency Response (Bode Plots).
2. Root Locus Plots.
3. Transient Response (Inverse Transform).
4. Inverse Root Locus.
5. Factoring Polynomials.
6. Sum of Two Transfer Functions.
7. Direct Transform of Time Function.
8. Loading Effect.
9. Carrier Systems.
10. Dead Time Systems.
11. Sampled Data Systems.
12. General Sampled Systems.
13. Power-Density Spectra.
14. Changes in Power-Density Spectra from Filtering or Passage Through a System.
15. Process Transfer Functions from Spectra.
16. Wiener-Hopf Equation Solution for Optimum System.
17. Non-Linear Systems by the Describing Function Technique.
18. Circuit Analysis.

© ELECTRO-MEASUREMENTS, INC. 1959
Litho in U.S.A.

esi

APPLICATIONS OF THE ESIAC ALGEBRAIC COMPUTER

1. Frequency Response (Bode Plots). Given: Pole-zero pattern of factored transfer function, log magnitude and phase versus log frequency can be plotted on two separate sheets.

   References: (see footnote) A. pp. 29-34, B. pp. 16-26, C. pp. 99-100.

2. Root Locus Plots. Given: Pole-zero pattern of open loop transfer function, on the log s plane can be plotted the locus of 180° phase shift, and the locus of a specified gain contour. The intersections are the closed loop pole locations. The closed-loop zeros are the open loop forward zeros and the feedback poles.

   References: A. pp. 221-250, B. Chapters 4 and 9, C. pp. 100-113.

3. Transient Response (Inverse Transform). The transient response of the closed loop system can be written down as the sum of exponential terms, one from each pole, with coefficients which are the residues at each pole. The magnitude and phase of each residue can be measured on the F scales when the pole at that point is removed. The measurements are made in sequence. For an inverse Laplace transform, all the transient terms are in positive time, and right-side poles are unstable. For an inverse Fourier, keeping the frequency axis vertical, right-side poles contribute negative-time terms which are stable but unrealizable as an impulse response, and which cease at zero time. This method can be used to convert a cross-power density spectrum into a cross-correlation function.

   References: A. pp. 26-29, B. Chapter 3, C. pp. 113-114.

4. Inverse Root Locus. Given: Desired pole-zero pattern of the closed loop system. To find: The required pole-zero pattern of the open-loop system. The open loop poles are at the zero db contours on the zero-degree root loci.

   References: B. Chapters 4 and 9.

Reference A: Truxal, J. G., Automatic Feedback Control System Synthesis, McGraw Hill Book Co., 1955.

Reference B: Smith, O. J. M., Feedback Control Systems, McGraw Hill Book Co., 1958.

Reference C: Evans, W. R., Control System Dynamics, McGraw Hill Book Co., 1954.

5. Factoring Polynomials. Given: A polynomial in s of the n-th degree, with the coefficients of each term. To find: The factors or roots. Six terms of the polynomial can be taken aside, set equal to zero, and the five roots found as points on a root locus plot. Three more terms can be added to this result, and the eight roots found with a second plot. This can be continued until all roots have been found.

   References: A. pp. 270-271, B. pp. 68-70, Bright, R. L., and G. H. Royer, Network Analysis by a New Semi-Automatic Computer, Carnegie Institute of Technology, Monograph.

6. Sum of Two Transfer Functions. Given: Two transfer functions, Required: Their sum, factored into zeros and poles. The poles are all of the poles of each function. The zeros are at the -1 points in the plot of the ratio of the transfer functions. This step is needed in minor loop feedback stabilization of a system.

   Reference: B. Chapter 4.

7. Direct Transform of Time Function. Given: Sum of exponential-like time terms. Required: Poles and zeros of transform. Method: Writing the direct transform as a residue divided by a pole factor, this may be cleared of fractions yielding an unfactored numerator divided by a factored denominator. The numerator zeros may be found by factoring as in Application 5.

   References: See Application 5.

8. Loading Effect. Given: Network function for infinite impedance output, and actual load resistance of input impedance of a following network. Required: Actual transfer function of the cascaded system. This is a modified root locus method. The poles are the roots of $1 + Z/R$, where Z is the driving point impedance of the unterminated network, and R is the terminating impedance.

   Reference: A. pp. 274-276.

9. Carrier Systems. Given: Transfer functions prior and following sinusoidal multipliers for modulators and demodulators. Required: Closed-loop poles and system stability. Method: The transform of the output of a modulator is the transform of the input shifted up by the carrier frequency. This sum has new zeros determined as in Application 6. The poles and zeros of the carrier frequency components are next superimposed, and the modulator operation is repeated for the demodulator. This loop transference may then be tested for stability as in Application 2.

   References: B. Chapter 18.
   Morris, David, "A Theoretical and Experimental Method of Modulation Analysis for the Design of A. C. Servo Systems". Automatic and Manual Control, A. Tustin, ed., Butterworth Scientific Publications, London, 1952.

10. Dead Time Systems. Given: Open-loop transfer function of the form G(s) H(z). Required: To find the closed-loop poles. Method: The function H(z) is converted to a function F(s) by any one of several approximations in the form

$$z = e^{-T_d s} \cong \prod \left(1 - \frac{s}{s_i}\right)^{n_i}$$

where s_i and n_i in a group of factors are chosen for the best approximation. The open-loop transfer function is then G(s) F(s) and the closed-loop poles are found by a root locus plot (Application 2).

References: A. pp. 546-553, B. Chapter 10.
Oldenbourg, R. C. and H. Sartorius, "The Dynamics of Automatic Controls", *A. S. M. E.* 1948, pp. 107-118.
Kuh, E. S., "Synthesis of Lumped Parameter Precision Delay Line", *I R E National Convention Record*, Circuit Theory P. G., 1957.

11. Sampled Data Systems. Given: Transfer function of open-loop unsampled system, and sampling frequency. Required: Closed-loop frequency response and stability test. Method: The computer is used to represent the z plane, where $z = e^{Ts}$. The location of each system pole and zero in the s plane is replotted in the z plane, using conformal mapping techniques. The frequency axis is now the line of constant $|z| = 1.0$, (not $j\omega$ as in the s plane). The open-loop frequency response is the magnitude and phase along the unity z line, where the angle of z is 2π times the ratio of the test frequency to the sampling frequency.

The closed-loop poles are at the 1 points on the 180° phase lines (root loci). They are unstable if they lie beyond a magnitude 1.0.

With the closed-loop poles and zeros entered in the z plane, the closed-loop frequency response is the magnitude and phase along the contour $|z| = 1.0$.

References: A. pp. 508-527.
Jury, E. I., "The Effect of Pole and Zero Locations on the Transient Response of Sampled-Data Systems", *A I E E Trans.*, Vol. 74 Part II, 1955, pp. 41-48.
Jury, E. I., "Correlation Between Root Locus and Transient Response of Sampled-Data Control Systems", *A I E E Trans.*, Vol. 74 Part II, Jan 1956, pp. 427-435.

12. General Sampled Systems. Given: Transfer function and sampling frequency of a system. Required: Closed-loop transfer function valid for an output which is not the output of the sampler, and for instants between the sampling instants. (Note: In Application 11, the results are valid only at the sampling instants, and only at the output of the sampler.) Method: The closed-loop poles are determined as functions of z as in Application 11. This function H(z) is converted to a function F(s) by approximation methods as in Application 10. Between the output of the sampler and the output of the system is a function G(s) which gives an open-loop transfer function of G(s) F(s). The closed-loop transfer function is then found by using a root locus plot.

This method is applicable to determining the inverse transform of any mixed distributed and lumped parameter function G(s) H(z). The residues at all of the poles are measured as in Application 3. The resulting time function is the valid closed-loop transient response.

References: A. pp. 508-546, see also references for Application 11.

13. Power-Density Spectra. Given: The poles and zeros of a power density spectrum in the Fourier λ plane. Required: A plot of magnitude and phase of this spectrum as a function of log frequency. The magnitude along the frequency axis is measured in the same manner as in Application 1. There will be poles and zeros in all four quadrants of the complex plane.

Reference: B. Chapter 5.

14. Changes in Power-Density Spectra from Filtering or Passage Through a System. Given: The poles and zeros of the spectrum and the transfer function of the system. Required: A plot of magnitude and phase of the spectrum of the system output. Method: To the pole-zero pattern of the spectrum is added the pole-zero pattern of the transfer function, and also the pole-zero pattern of the complex conjugate of the transfer function (mirror image in the frequency axis). This resultant pattern is the spectrum of the output. The magnitude is read along the frequency axis. The phase is zero degrees if the self-power spectrum is used.

For the cross-power spectrum between the system output and another signal, to the pole-zero pattern of the cross-power spectrum between the system input and the other signal is added the pole-zero pattern of the complex conjugate of the transfer function. The output cross-power spectrum will have the phase as measured along the frequency axis.

References: B. Chapter 5.
Goldman, Stanford, Information Theory, Prentice Hall Book Co., pp. 278-279.

15. Process Transfer Functions from Spectra. Given: An operating process and measured spectra at the input and the output of the controller, for two different controller gain settings. Required: The open-loop and closed-loop process transfer functions, and the spectrum of the equivalent load disturbance at the process output. Two or more root locus plots are required.

Reference: Smith, O. J. M., Discussion on "Correlation Functions and Noise Patterns in Control Analysis", by H. Thal-Larsen, Trans. A. S. M. E., August 1957.

16. Wiener-Hopf Equation Solution for Optimum System. Given: Spectra of signal and equivalent noise at the system input, and the transfer functions of unalterable system components. Required: The open-loop transfer functions of the optimum control system. Method: The input spectrum of the sum of the signal and the noise is determined using Application 6 above. This is factored into the realizable and unrealizable parts, and the pole-zero pattern for $\frac{\phi_{is}}{\phi_{ii}^{-}}$ is introduced into the computer.

The realizability operator is applied to this pattern by measuring the residues at the realizable poles in the computer. This operator is the Laplace of the inverse Fourier, $\mathcal{L}\mathcal{F}^{-1}$.

The $\mathcal{L}\mathcal{F}^{-1} \frac{\phi_{is}}{\phi_{ii}^{-}}$ consists of a sum of poles with residues, which, when cleared of fractions, is an unfactored numerator divided by a factored denominator. The numerator should be factored, using Application 5, with the resultant closed-loop system design given by

$$\frac{1}{\phi_{ii}^{+}} \mathcal{L}\mathcal{F}^{-1} \frac{\phi_{is}}{\phi_{ii}^{-}}$$

To find the open-loop transfer function, Application 4 may be used. To include the unalterable components, a predictor-like formulation of the optimum system is used.

References: B. Chapter 6.
Smith, O. J. M., "Separating Information from Noise", Trans. I R E, Prof. Group on Circuit Theory, PGCT-1, Dec., 1952, pp. 81-100.
Goldman, Stanford, Information Theory, Prentice Hall Book Co., pp. 256-277.

17. Non-Linear Systems by the Describing Function Technique: Given: The open-loop transfer function of the linear parts of the circuit, and the describing function for the non-linear parts of the circuit. Required: Closed loop stability and Gain and Phase plots of the fundamental component of the output for various input signals.

References: A. pp 559-612, B. Part III

18. Circuit Analysis: Given: An electrical circuit. Required: The pole-zero plot for the circuit and the phase and gain plots to study frequency responses. The root locus technique is used to factor polynomials encountered in transfer functions and immitances, and to obtain the closed loop response of a feedback circuit from its open loop response.

References: Stewart, J.L. Circuit Theory and Design, Wiley 1956
Angelo, E.J. Electronic Circuits, McGraw-Hill, 1958
Van Valkenberg, M.E. Network Analysis, Prentice Hall, 1955